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ABSTRACT

An extension of the Bock-Samejima model for multiple choice items is introduced. The model provides for varying probabilities of the response alternative when the examinee guesses. A marginal maximum likelihood method is devised for estimating the item parameters, and likelihood ratio tests for comparing more and less constrained forms of the model are provided. Applications of the model are illustrated with item response data for the word knowledge and general science subtests of the Armed Services Vocational Aptitude Battery.

A Response Model for Multiple Choice Items

In practical ability measurement, binary item response models have been applied routinely to data from multiple-choice tests with four or five alternatives per item. To use the binary models, the data have been dichotomized (correct and incorrect) and the distinct identity of the incorrect alternatives has been lost. This procedure logically follows the tradition of "scoring" a test by considering only the correct answers; however, the binary models are incomplete as theories of the item responses as useful information may be lost in the dichotomization.

It is possible to conceive of a unidimensional latent variable model which completely explains the item response data, and several have been proposed. The model which will be introduced here is an extension of that proposed by Bock (1972) and subsequently extended by Samejima (1979); a different parameterization of the same model has been discussed by Sympson (1983). Bock (1972), Thissen (1976), Sympson (1983), and Levine and Drasgow (1983) have shown that some increased precision of measurement may be obtained when information in the incorrect alternatives on multiple choice tests is included in the item response model. In a subsequent section, we will discuss this gain in information with a new model, as well as some attendant difficulties. In the next section we will introduce the model and describe a workable scheme for maximum likelihood estimation of its parameters.

The Model

The model for multiple-choice items to be described makes heavy use of the multivariate logistic transformation suggested as an item response model by Bock (1972), so it is useful to begin with a clear understanding of that model. For m categorical responses, we specify m response functions $z_k = a_k\theta + c_k$, each of which is a linear function of the latent ability variable θ. Usually $z_{correct}$ would have a positive slope and the other z_k's would have lesser positive or negative slopes. The linear functions describe one of the simplest possible relationships between the response and θ. To make this a model for categorical item responses, the z_k (which lie on the real line) must be mapped into [0,1]. This is accomplished by the multivariate logistic transformation, so

$$P(x_j=h|\theta;\underline{a},\underline{c}) = \frac{\exp(z_h)}{\sum_{k=1,m_j} \exp(z_k)} \quad . \quad (1)$$

The function (1) is Bock's (1972) model for an item response, $x_j=h$, in which $h = 1,2,\ldots m_j$ for a multiple choice item j with m_j (classes of) response alternatives. The item parameters are the vectors $\underline{a}$ and $\underline{c}$, subject to two suitable linear constraints (see below), giving $2(m_j-1)$ free parameters. The model described by (1) is moderately flexible, but lacks flexibility in certain crucial respects at the extremes. Specifically, one of the response alternatives must have the largest positive value of a_k; the trace line for that alternative is then monotonic increasing. That may be theoretically acceptable, since it

is probably the correct response. However, another alternative must have the largest negative value of a_k ($\Sigma a_k=0$ is one of the linear constraints), and that alternative's trace line must be monotonic decreasing. The latter aspect of the model is less acceptable: it implies that as ability decreases the probability of selecting one particular incorrect response approaches unity, and all the others go to zero. That is unlikely.

Samejima (1979) proposed a solution to this problem in a conceptual modification of Bock's (1972) model in which she added a completely latent response category labelled "zero." Here, we will sometimes refer to this category as "don't know" (DK). Lord (1982) has introduced a similar conceptual entity which he describes by saying the examinee is "(totally) undecided." In Samejima's model, DK is not an observed response, but a latent one, multi-logit-linearly dependent on a latent trait (giving two layers latent); the idea was that some proportion d_h ($=1/m_j$ in Samejima (1979)) "guessed" each of the observable response alternatives and were "mixed in" with the examinees who chose those alternatives intentionally. So the model becomes

$$P(x_j=h|\theta;\underline{a},\underline{c},\underline{d}) = \frac{\exp(z_h)}{\sum_{k=0,m_j}\exp(z_k)} + \frac{d_h\ \exp(z_0)}{\sum_{k=0,m_j}\exp(z_k)} ,$$

in which the second term adds a proportion d_h of those who "don't know" into each trace line; call it $P_j(h)$ for future brevity:

$$P_j(h) = \frac{\exp(z_h) + d_h\ \exp(z_0)}{\sum_{k=0,m_j}\exp(z_k)} , \qquad (2)$$

in which h takes the values $1,2,\ldots m_j$. Samejima's (1979) model had two more parameters than Bock's (1972) model: a_0 and c_0. In her presentation, the d_h were fixed and equalled $1/m_j$; this represents the hypothesis that those of sufficiently low ability assign their responses randomly with equal probability to each of the response alternatives. We found that unlikely; later we will show that it is not empirically the case. So we extend the version of the model given in (2) to allow the d_h, $h=1,2,\ldots m_j$ to be functions of estimated parameters.

Indeterminacies and Constraints

The model expressed in (2) has a number of indeterminacies and requires the imposition of some constraints to become identifiable. The linear constraints required on $\underline{a}$ and $\underline{c}$,

$$\sum_{k=0,m_j^k} a_k = \sum_{k=0,m_j^k} c_k = 0$$

are imposed by reparameterization:

$$\underline{a} = T\underline{\alpha}$$

and

$$\underline{c} = T\underline{\gamma} \quad ,$$

where T is an $(m_j+1) \times m_j$ transformation matrix, and $\underline{\alpha}$ and $\underline{\gamma}$ are vectors of free parameters of order m_j; T is the transpose of that in Bock (1972) throughout. There is a further indeterminacy with respect to the sign of $\underline{\alpha}$; this is an indeterminacy of reflection of the latent variable

θ which is logically identical to the rotational indeterminacy of common factor analysis. The reflection indeterminacy is not explicity solved, but rather the iterative solution of the likelihood equations is started with a_h positive for h = the correct response; this is usually sufficient to keep the estimated solution "right-side-up."

Since the vector $\underline{d}$ represents a set of m_j proportions, the d_h must be constrained to sum to unity and lie on the interval [0,1]. The dual constraint is imposed in two parts. In the first, the same multivariate logistic used in the model is also employed to make the d_h proportions from a set of psuedo-parameters d_h^*, which lie on the real line:

$$d_h = \exp(d_h^*) \; / \sum_{k=1,m_j} \exp(d_k^*) \quad . \qquad (3)$$

The parameters d_h^* are on the real line, but like $\underline{a}$ and $\underline{c}$, they must satisfy $\Sigma d_k^* = 0$. So

$$\underline{d}^* = T_2\underline{\delta} \qquad (4)$$

in which T_2 is an m_j x (m_j-1) transformation matrix of the same form as T (above) but of lesser order, and $\underline{\delta}$ is a vector of length (m_j-1) of parameters which (through (4) and (3)) give the proportions d_h.

So the set of free parameters for an item consists of m_j α_k's ("a-contrasts"), m_j γ_k's ("c-contrasts"), and (m_j-1) δ_k's ("d-contrasts"), for a total of $3m_j-1$ parameters: 11 for a four-alternative multiple-choice item.

The estimation system we use permits any of the elements of $\{\underline{\alpha},\underline{\gamma},\underline{\delta}\}$ to be fixed at any constant value, or to be constrained to be

equal to any other parameter. With this facility, our model (2) includes the previous models hierarchically as subsets. If $\underline{\delta}$ is fixed at $\underline{0}$, than all the $d_h=1/m_j$ and (2) becomes Samejima's (1979) model. If the parameters a_0 and c_0 are fixed at zero, the d_h become irrelevant (and are deleted mechanically) and the model becomes Bock's (1972) model. In our examples, we will compare the goodness of fit of all of these models, as well as versions of (2) that include equality constraints on particular parameters imposed across items.

The Relationship of the Model to Data

Models in item response theory (IRT) are intended to explain observed covariation among test-item responses. Unidimensional "latent trait" theories explain that covariation by appeal to an underlying latent variable (usually denoted θ) on which the probabilities of the responses are functionally dependent. Each pair of item responses is theorized to have non-zero covariance because the probability of both item responses depends on θ. The models usually hold that θ is the sole cause of covariation among the item responses; conditional on θ, the item responses are theorized to be independent ("local independence").

A complete IRT model for the item response data for a multiple-choice test in which n items each have m alternatives would be a model for the covariation in the m^n contingency table containing the counts of respondents giving each response pattern. Such a 4^2 table for two

four-alternative multiple-choice items is given in Table 1.

Insert Table 1 About Here

Evidence that there is some information in the examinees' incorrect choices is apparent (or at least can be found) in the covariance in the table. (The column percentages for the counts are given in parentheses to aid in interpretation.) Alternative C is correct for item 2; respondents who select C on item 2 are most likely to choose B on item 1; B is correct, so that is double evidence of their high ability. But examinees who choose A or B on item 2 are more likely to respond correctly to item 1 (36% and 34%) than those who pick D on item 2 (only 25%). A possible explanation is that those who select A or B on item 2 are higher on the ability continuum than are those who pick D. Even with 16 cells, this sort of argument may be lengthy, but the idea is that wrong responses are related to each other; if the responses are locally independent, this implies a relationship to "the trait."

The goodness of fit of a multiple category IRT model to the data in such a table may be evaluated with conventional likelihood ratio χ^2 statistics. For sets of data with more examinees than cells in the table, the likelihood ratio test against a general multinomial alternative may be used. For larger tables (35 four-choice items gives a 4^{35} table), there is no "general multinomial alternative"; but the likelihood ratio test between hierarchically nested models still may be used to evaluate the significance of the additional parameters of the

larger model.

Estimation

In general, the data for the estimation of the parameters $\{\underline{\alpha},\underline{\gamma},\underline{\delta}\}_j$ for each item j, j=1,...,n consist of the counts of response patterns $r_{\underline{x}}$ (where each x_j may take values in $[1,2,\ldots m_j]$) in an $m_1 \times m_2 \times \ldots \times m_n$ contingency table like Table 1. The probability of observing a particular response pattern $\underline{x}$ when drawing an examinee from a population in which θ follows a distribution $\phi(\theta)$, assumed N(0,1) in the examples, is

$$P(\underline{x}) = \int_{-\infty}^{\infty} \prod_{j=1,n} P_j(x_j)\ \phi(\theta)\ d\theta \qquad (5)$$

in which $P_j(x_j)$ is from equation (2).

The likelihood for the entire set of observed data is

$$L = C \prod_{\underline{x}} P(\underline{x})^{r_{\underline{x}}} \qquad (6)$$

in which C is a normalizing constant not dependent on the parameters, and the product runs over all possible response patterns. In practice, of course, for more than a handful of items the number of possible response patterns is astronomical, and the "count" in each cell is one or zero. In such cases, only those response patterns actually observed need to be considered in the computation of (6), or (7), a function proportional to the loglikelihood:

$$\ell_o \sim \sum_{\underline{x}} r_{\underline{x}} \log P(\underline{x}) \quad . \qquad (7)$$

For a very small number of items (3 or 4; the parameters of the full model are not identifiable with only 2 items, as there are fewer cells than parameters), it may be possible to obtain the MLEs by directly maximizing (7) with a Newton-type algorithm. Bock (1972) reported such results for his original model with procedures similar to those used by Bock and Lieberman (1970).

Bock and Aitkin (1981) described an extremely simple and elegant algorithm for maximizing functions like (7) for binary models, and we extend that algorithm to the estimation of multiple category models. The Bock-Aitkin algorithm is a two-step procedure like the "EM-algorithm" (Dempster, Laird, and Rubin, 1977), so we describe first the "E-step" and then the "M-step." The procedure is iterative, and EM-pairs are repeated until the process converges.

The E-step

The Bock-Aitkin algorithm is based on a discrete representation of $\phi(\theta)$ and the integrand of $P(\underline{x})$, both continuous densities, over Q "quadrature points" θ_q, with q=1,2,...Q. Such a discrete representation of the continuous densities may be made arbitrarily close to continuous reality by choosing Q large, just as numerical integration may be made arbitrarily accurate by using sufficient quadrature points. However, large values of Q slow the computations; Q=10 over the range θ=-4.5 to θ=4.5 in unit steps seems to be sufficiently accurate and fast for many applications and is used in all of the examples here.

Under the assumption that the population is composed of individuals who are members of Q discrete "classes" with values θ_1, θ_2, ..., θ_Q on the latent variable, "complete data sufficient statistics" for the estimation of the item parameters $\{\underline{\alpha},\underline{\gamma},\underline{\delta}\}_j$ for item j would consist of a table of counts R_j^*, in which each element r^*_{jkq} is the number of individuals in class θ_q selecting response alternative k on item j. So the E-step of the Bock-Aitkin algorithm consists of computing the expected values of the r^*_{jkq}, conditioned on the data and the current provisional estimates of the item parameters.

Using provisional estimates of the item parameters for each item (as starting values we use $\underline{\delta}=\underline{0}$, $\underline{\gamma}=\underline{0}$, and $\alpha_k=1$ for categories k which are incorrect and $\alpha_k=2$ for category k correct), compute the elements of the m_j x Q table R_j^* containing:

$$E(r^*_{jkq} \mid \text{data}; \{\hat{\underline{\alpha}},\hat{\underline{\gamma}},\hat{\underline{\delta}}\}) = \sum_{(\underline{x} \text{ in } \xi)} r_{\underline{x}} [P(\underline{x};\theta_q)/\Sigma_q P(\underline{x};\theta_q)] \quad (8)$$

in which ξ is the set of $\underline{x}$ in which $x_j=k$ and

$$P(\underline{x};\theta_q) = \prod_{j=1,n} P_j(x_j)\phi(\theta_q) \quad . \quad (9)$$

Note that, while (9) and (8) are computed in a (potentially long) loop over the observed response patterns, the values $P_j(k)$ for each item from (2) are required only for a fixed set of Q values of θ. If those values are placed in a table before the E-step is begun, the computations involved in (8) and (9) are limited to table look-up, multiplication, and addition. The E-step yields a set of n m_j x q tables of non-integral artificial "counts" which are used as data in the M-step.

The M-step

The M-step consists of maximum likelihood estimation of the parameters $\{\underline{\alpha},\underline{\gamma},\underline{\delta}\}_j$ for all items j=1,2,...n, using the tables of expected values R_j^* as data. It is simply nonlinear regression.

In terms of the "data" in R_j^*, the loglikelihood for item j is

$$\ell_j \sim \sum_k \sum_q r^*_{jkq} \log P_j(k;\theta_q) \qquad (10)$$

in which $P_j(k;\theta_q)$ is equation (2) evaluated at θ_q. Standard gradient methods may be used to maximize ℓ_j over the parameter space. The MLEs of the parameters are obtained where

$$\frac{\partial \ell_j}{\partial \zeta} = 0$$

for all parameters in $\zeta=\{\underline{\alpha},\underline{\gamma},\underline{\delta}\}$. Since for any set of parameters constrained by T such that $\kappa=T\zeta$,

$$\frac{\partial \ell_j}{\partial \zeta} = T \frac{\partial \ell_j}{\partial \kappa} \quad , \qquad (11)$$

we will state the required components of the gradient for the constrained parameters $\{\underline{a},\underline{c},\underline{d}\}$; the gradients used to maximize (10) are then given by (11). Further,

$$\frac{\partial \ell_j}{\partial \kappa} = \sum_k \sum_q r^*_{jkq} \frac{1}{P_j(k;\theta_q)} \frac{\partial P_j(k;\theta_q)}{\partial \kappa} \quad ; \qquad (12)$$

So what we really need are the derivatives of (2) with respect to $\underline{a}$, $\underline{c}$, and $\underline{d}^*$ (substitute all for κ above). To write these, a useful bit of

shorthand is

$$e_k = \exp(a_k\theta + c_k) \quad ,$$

which is always assumed to be evaluated at the appropriate value of θ; all summations of e_k are over categories $k=0,1,\ldots m_j$. The elements of the gradient (P_h is short for $P_j(h;\theta_q)$ hereafter) to be substituted in (12), then (11), and finally zeroed to maximize (10) are:

$$\frac{\partial P_h}{\partial a_0} = \frac{[(\Sigma e_k)\theta d_h e_0 - (e_h + d_h e_0)\theta e_0]}{(\Sigma e_k)^2}$$

$$\frac{\partial P_h}{\partial a_h} = \frac{[(\Sigma e_k)\theta e_h - (e_h + d_h e_0)\theta e_h]}{(\Sigma e_k)^2}$$

$$\frac{\partial P_h}{\partial a_{h'}} = \frac{-\theta e_{h'}}{\Sigma e_k}$$

$$\frac{\partial P_h}{\partial c_0} = \frac{[(\Sigma e_k)d_h e_0 - (e_h + d_h e_0)e_0]}{(\Sigma e_k)^2}$$

$$\frac{\partial P_h}{\partial c_h} = \frac{[(\Sigma e_k)e_h - (e_h + d_h e_0)e_h]}{(\Sigma e_k)^2}$$

$$\frac{\partial P_h}{\partial c_{h'}} = \frac{-e_{h'}}{\Sigma e_k}$$

$$\frac{\partial P_h}{\partial d_h^*} = \frac{e_1}{\Sigma e_k} \cdot \frac{\Sigma d_k^* - d_h^{*2}}{(\Sigma d_k^*)^2}$$

The gradient does not simplify very much since the model is not part of an exponential family. However, it is possible to locate the maximum of ℓ_j using these derivatives, and use the resulting set of parameters in the next E-step. Our estimation procedure allows equality constraints to be placed across as well as within items; this requires some of the gradients to be accumulated across items, but causes no serious problem beyond book-keeping. We use a conditioned Newton-type algorithm (MINIM, described by Haberman, 1974) to locate the maximum of (10); the conditioning is useful since the matrix of second derivatives may be nearly singular for some items.

Convergence and Local Minima

Using the updated item parameters from the M-step, the sequence [E-step, M-step] is repeated until either a) the parameters stabilize or b) a fixed number of cycles is reached (we usually use 15 or 20). The algorithm sometimes converges in the parameter space. More often, it does not, but the lack of convergence causes no real problems. In such cases, most of the parameters (and the loglikelihood) remain roughly constant after ten or fifteen cycles, while a parameter (or a few) change linearly and indefinitely. The changing parameter is frequently a particular α_k (and its associated γ_k), changing as the associated a_k rises toward a very high (possibly infinite) MLE. The difference in goodness of fit with such a slope high (3. to 5. or so) and

much higher appears to be negligible, so there is no loss in simply stopping the estimation procedure arbitrarily. In such a case, the trace line is fairly well-determined; but the numerical value of the associated a_k is not.

A second case in which the parameters fail to clearly converge arises with γ_k's associated with a_k's near zero. The parameter c_k becomes ill-defined when a_k is zero, and the associated γ_k "wanders." Again, there is no loss of fit if the estimation procedure is stopped. The Bock-Aitkin algorithm may be speeded slightly using acceleration as in Thissen (1982), but the value of the acceleration parameter should be limited to about -1, which doubles the step-size at each cycle, to avoid oscillation.

The likelihood surface for the model clearly has more then one "local" minimum. The indeterminacy with respect to reflection of θ gives two equal, identical modes. With poorly chosen starting values, the estimation procedure has located other (apparent) stationary points; usually they seem to be located in a peculiar region corresponding to some items using both reflections of θ in the same solution -- these may be modes or saddle points. Good starting values and the EM-like nature of the Bock-Aitkin algorithm provide a solution in this case, because EM-algorithms only climb local modes. If the algorithm starts near the desired part of the likelihood surface, it will end there. It appears that sensible starting values solve the problem.

The likelihood surface is multi-modal and may be ill-behaved, producing parameter estimates that are sometimes undefined or are on the

boundary. The entire system begs for full Bayesian treatment, with a prior distribution restricting the parameters to a reasonable part of the space under all circumstances. At this stage however, it is not clear what sort of prior may be appropriate. After some experience with the model is gained beyond that in the examples which follow, we may be in a position to specify reasonable parameters for a prior.

Alternatively, a different model, in the exponential family and with a unimodal likelihood (preferably with similar properties to this one in terms of fitting the data), would also solve all of these problems. Such a model has yet to be proposed.

Characterizing $\underline{\theta}$

The parameter estimation described above usually has the goal of "calibrating" a set of test items, after which the parameters are to be taken as known and used to characterize θ for examinees who produce a particular response pattern $\underline{x}$. Given a set of item parameters, the posterior density for θ is

$$P(\underline{x};\theta) = \prod_{j=1,n} P_j(x_j)\phi(\theta) \quad , \qquad (13)$$

in which $P_j(x_j)$ is from equation (2). If the model is correct, (13) describes the distribution of examinees who respond with pattern $\underline{x}$. It can be characterized graphically, and two examples will be presented below in Figure 5.

For more than a few items, (13) is roughly Gaussian in shape, and so it may also be described by estimates of its location and spread. An

extension of the procedure commonly used in binary IRT is to use the mode as an estimate of the location of (13), where

$$\frac{\partial \log P(\underline{x};\theta)}{\partial \theta} = \sum_{j=1,n} \frac{\partial \log P(x_j;\theta)}{\partial \theta} + \frac{\partial \log \phi(\theta)}{\partial \theta}$$

is zero, with the variance approximated by the negative inverse of

$$E \frac{\partial^2 \log P(\underline{x};\theta)}{\partial \theta^2} .$$

The modal estimate is practical to compute as long as $\phi(\theta)$ is a reasonable function, and easy if $\phi(\theta)$ is normal. There is no guarantee that (13) is unimodal; for small numbers of items it is likely to have more than one mode. Multimodality presents potential problems for mechanical use of modal estimates.

However, it is also fairly straightforward to numerically integrate (13) to obtain its mean and variance. The mean has been called the "EAP" (Expected A Posteriori) estimate of θ by Bock and Mislevy (1982). An advantage of the EAP procedure over modal estimation is that the derivative of $\phi(\theta)$ is not required; therefore $\phi(\theta)$ may take any form describable as a histogram with finite variance.

Examples

To illustrate application of the model to item response data, we have analyzed several subsets of items from the ASVAB Form 8A administered to a national probability sample of youth by the National Opinion Research Center; the data are described by Bock and Mislevy

(1981). The data used here are from a 10% subsample of the NORC sample, total N=1178. For small illustrations of the performance of the model, we use four-item subsets consisting of the last four items of the Word Knowledge (WK) and General Science (GS) sections of the test. Four items were used because, with roughly a thousand examinees, that is the largest number for which the number of cells in the m^n table is less than N (4^4=256), so significance tests of the overall goodness of fit of the model against a general multinomial alternative are feasible. The last items were used because they are most difficult, giving the trace lines more latitude to be fitted.

For the four-item examples, only examinees with complete data (no non-response) for all four items were used. For GS, N=1048 and 214 of the 256 cells of the table were filled (one of the 4 x 4 marginal tables of this table makes up Table 1); for WK, N=976 and only 156 response patterns were observed.

We also fitted several forms of the model to the entire set of 35 WK items, and a subset of 12 of the 25 GS items called "physical science" hereafter. (The ASVAB "General Science" section consists of about half biological and half physical science questions; we selected a subset more likely to be unidimensional.) In these analyses, all examinees were used (N=1178) and non-response data to individual items were ignored (not placed in any category).

Results

Table 2 gives summary goodness of fit results for the four-item examples.

Insert Table 2 About Here

Bock's (1972) model and Samejima's (1979) modification are both rejected at the $p<0.05$ level for both sets of data. The model called "ABCD" is a form of (2) in which the vector $\underline{\delta}$ is estimated, but restricted to be equal across items: this represents the hypothesis that a (constant) proportion of those who "don't know" select alternative A, a different proportion B, and so on; but the distribution over A, B, C, and D (regardless of the "correct" response alternative) is the same for all items. For the WK data the model is a great improvement over Samejima's $\underline{\delta}=\underline{0}$ model ($G^2=12.6$ on 3 d.f.), and for GS it is not. The estimated vector $\underline{d}$ for WK for [A,B,C,D] is [.1,.2,.4,.3].

In the last four WK items, D is correct for the first and the last and C is correct for the others; so to test the hypothesis that (somehow) the correct alternative "attracts" guessing we tested "ABCD(C),ABCD(D)" in which the two pairs of items had different vectors $\underline{\delta}$. This model fits the WK four-item data, and the improvement over ABCD is significant ($G^2=10.6$, d.f.=3, $p<0.02$); the parameter estimates are given in table 3. The "saturated" version of (2), with 11 free

parameters per item, does not fit the WK data significantly better.

Insert Tables 3 and 4 About Here

The last four GS items do not have such a convenient structure of correct alternatives; but for these items "correctness" seemed more important than alternative position, so in the model "correct vs. incorrect" d_h is constrained to be one value (the same for all items) for the correct response and another for all the incorrect responses. This model (barely) fits the GS data. The parameter estimates are given in Table 4. Forty percent of the DK guess the correct alternative; there is no evidence that this varies across items (The last line of Table 2 gives the G^2 for "correct vs. incorrect each item").

When the complete WK data are fitted, the likelihood ratio test against a "general multinomial alternative" for the 1178 observations in a 4^{35} table is, of course, meaningless. Nevertheless, we report likelihood ratio tests between variously constrained versions of the model. The Samejima (1979) model is the most constrained form that we use for comparison. A model of the "ABCD" form of (2), in which $\underline{\delta}$ is estimated, equal across items, gives a reduction $G^2=41$ on 3 d.f., $p<0.001$; the estimate of $\underline{d}$ was [.2,.3,.3,.2] for [A,B,C,D]. As with the four-item WK, a model in which $\underline{\delta}$ is constrained to be equal only among items with the same (letter) correct response fits significantly better: $G^2=72$ on 9 d.f., $p<0.01$. No more complex models were considered, as estimation of 11 parameters per item for the very easy items (to which

the majority of examinees responded correctly) would be estimating many parameters with little data. The estimated parameters for the last four items obtained in the analysis of the entire WK set are given directly below their values for the four-item estimation in Table 3.

Analysis of the physical science portion of the GS section gave similar results: (2) in its "ABCD" form fitted significantly better than Samejima's model, $G^2=29$ on 3 d.f., $p<0.01$. The estimate of $\underline{d}$ was again [.2,.3,.5,.2]. And the model in which $\underline{d}$ varied according to the letter-value of the correct alternative was better: $G^2 = 26$ on 9 d.f., $p<0.01$. Two of the items in the "physical science" set are among the last four GS items, and their parameter estimates are given in Table 4.

Discussion

In cases in which the fit can be tested -- with a thousand examinees and four-alternative items, four items -- some forms of the model given by (2) and (5) fit item response data satisfactorily. This represents a major step forward in item analysis, because it is no longer necessary to look at deviations from the fit to examine items; we may examine the fit itself.

Insert Figure 1 About Here

The solid curves in Figure 1 illustrate the trace lines for the "physical science" fit to GS item 23, which is item 2 in Table 1, discussed above. From Table 1 we inferred that those who chose A or B

might be more able than those who chose D. Indeed, the trace line for D restricts that response to those with values of $\theta<0$ for all practical purposes, while B-responses come mostly from those with $\theta>0$, and A is spread all over. The DK trace line indicates that most of those below $\theta=-.5$ have no idea; and more of them guess correctly than any other way.

The dashed curves in Figure 1 illustrate the trace lines for the "four-item" fit to item 23. Note in Table 4 that the parameter estimates differ a great deal, but the solid and dashed trace lines in Figure 1 differ much less (except for DK) in the middle. DK is a completely inferred latent response and is not really very well-defined by four items. The other curves are more similar in the middle, where the data are; 95% of the population distribution lies between θs of -2 and +2. The different parameter estimates seem to affect the curves primarily at the extremes.

Unlike algorthms which make use of point-estimates of θ in the estimation of item parameters, and require both large numbers of examinees and large numbers of items, the MML estimation procedure used here should be consistent considering the number of examinees alone if the model is correct. That is, as the number of examinees becomes large the estimates for four items or 12 or 35 should all converge to the true values. So the dashed and solid lines in Figure 1 should be the same. They are very similar, and there are three possible reasons for the small differences observed between them. First, one thousand examinees may easily not be "asymptotic" for a table with 256 cells. Second, one or the other solution may not be completely converged; EM-algorithms can

be slow near the maximum. Third, and most likely, the model may be incorrect in that the latent dimension θ may be defined slightly differently by the most difficult four items than by the entire test.

Insert Figure 2 About Here

Figure 2 similarly illustrates the four-item and complete test estimates of the trace lines for WK 32, which demonstrates this effect more graphically. With estimates from the entire test, DK goes to unity for low θ. But the four-item (dashed) curve for DK rises for moderately low θ and then goes back down, and A goes to unity for low θ. That can't be right; but it is all happening in the region below θ=-2, where there are essentially no data and the model is extrapolating. Extrapolation is bad: the model is too flexible at the extremes. Flexibility was one of the goals of the model, but it seems we may have overdone it. This is in marked contrast to traditional IRT models which go to unity, zero, or some asymptote at the extremes, and never misbehave there. Note that even with all of this strangeness, the other trace lines in Figure 2 are essentially identical between θs of -2 and +2.

Insert Figure 3 About Here

Figure 3 shows the trace lines for the correct responses only for WK items 32-35 from the four item set and the whole test, as well as

traditional 3-PL curves estimated for the entire test (dotted lines) for comparison. If the "tail-wagging" to the left of θ=-2 is ignored, the pairs of curves for the whole test are nearly identical and the four-item curves are somewhat deviant. That may be capitalization on chance in four items -- or it may be that θ differs mildly in the hardest items of the test from its definition in the entire test. None of the curves are extremely different.

Insert Figure 4 About Here

For WK as a whole, the usual result (of Bock, 1972; Thissen, 1976) is obtained with respect to test information: Figure 4 shows the information curves for the multiple category scoring and the binary scoring (3-PL). For θ<0, information from incorrect responses increases total information by about 50%, equivalent to extending the test (for half the examinees) from 35 to 50 items.

Insert Figure 5 About Here

Figure 5 illustrates the process. The solid curves show the trace lines associated with a particular set of responses [B,B,B,B], all incorrect, for the last four WK items. Then the N(0,1) population distribution is plotted with the the product of all five curves, or posterior density, labelled "Total." The mode of that density is traditionally $\hat{\theta}$; it is about -0.7 (s.e.=0.6) in this case. The dashed

curves show the 3-PL trace lines, all incorrect, so they are all monotonic decreasing. That is not very informative; the (dashed) posterior is broader (s.e.=0.7) and closer to zero ($\hat{\theta}$=-0.5) because there is less information so the estimator is "shrunk" more toward the mean of the population distribution. If the model approximates the world reasonably well, the multiple category scoring gives more information about people responding [BBBB] than can 3-PL.

General Discussion

The model proposed here provides the first practical, complete IRT item analysis for multiple choice tests, describing the performance of all of the response alternatives as functions of the trait being measured. The parameters of the model are not readily interpretable for the most part, but graphical presentation of the trace lines and/or their products with the population distribution gives thorough item analysis which has much to recommend it.

This sort of item analysis is sufficiently flexible that "bad" trace lines are fitted. These can then be observed and such items modified or eliminated in the course of empirical test development. That is not possible, in general, with simpler models which require examination of the residuals to find bad items. For item analysis, the model of (2) and (5) is clearly an excellent choice; for scoring tests (aka "estimating $\hat{\theta}$"), matters are more complicated.

Non-monotonic trace lines for the correct response have become a popular feature of recent IRT developments: Lord (1982) and Choppin

(1983) have proposed item response models which permit "low-ability" non-monotonicity, in work on binary models independent from Samejima's (1979) multiple category proposal. There are a number of possible explanations for non-monotonicity on the left, as for the correct response in Figure 1. The only requirement for candidacy as an explanation for the effect is that it must account for "getting the right answer for the wrong reason." Bock (personal communication, 1983) suggests a name for the phenomenon: "positive misinformation."

Two sources of positive misinformation come to mind. The first is that the correct response for a particular item differs from the distractors on dimensions other than that which is intended and observable, given sufficient ability. Examinees of medium and high ability perceive the features and attempt the processing intended by the item-writer, while examinees of low ability see other features of the alternatives which cause them to select the correct one. These "other features" may be effectively "invisible" to individuals of higher ability, and therefore to the item-writers as well.

A second possible cause of non-monotonicity in the correct trace line is cheating. If low-ability individuals cheat (e.g., by copying a neighbor's answer, which is more likely to be correct than not for most items), then the resulting correct alternative trace lines will rise for those of very low ability.

It may also be possible that non-monotonic trace lines are estimation artifacts in small sets of items (like four). It may be that, with few monotonic correct-alternative trace lines to "orient" θ, the estima-

tion procedure could "wrap around" and (effectively) place some of the high ability individuals on the left end of the θ-continuum. But this explanation can be discounted with longer tests, as the non-monotonicity there comes from correct response to an item being more likely from examinees who get most of the other items wrong than from those who get fewer of the other items wrong.

Distinguishing among these possibilities would require experimentation with the items and the testing situation. The model does, however, offer an item analysis which permits such phenomena to exhibit themselves in the fit, when they are present.

So the use of non-monotonic curves in item analysis may be required to fit the observed data. However, the use of non-monotonic trace lines in constructing the posterior density, a measure of the central tendency of which is to be called $\hat{\theta}$ and used to "score the test", gives rise to many potential problems. If the correct response trace line for item j is non-monotonic on the right (e.g., it turns down), then examinees with some response patterns on the other items will be "penalized" by responding correctly to item j; they would have been assigned a higher $\hat{\theta}$ if they had selected certain of the incorrect alternatives. Correct response trace lines which are non-monotonic on the left similarly "penalize" examinees of low ability who respond correctly. As measurement, this is all probably satisfactory: conditional on the other item responses, a correct response to item j may not imply higher ability; it may be more likely to be guessing or cheating. But this may be a problem for the test "as contest." Further, it might be difficult

to defend such a method of test scoring against the onslaught of members of the bar before a jury. A case can, therefore, be made for either a) rejecting the model or b) rejecting the use of items with non-monotonic trace lines for the correct response.

Multiple category scoring of this nonlinear sort has certain bizarre properties even if non-monotonic trace lines are eliminated. For certain regions on the θ-continuum, selecting a particular incorrect response will increase $\hat{\theta}$ more than would selecting the actual correct response. For instance, in the Word Knowledge item in Figure 2, for θ just below -1, selecting response B will increase $\hat{\theta}$ more than would selecting the correct response. But for θ around 1, selecting B will "penalize" the respondent! The contingencies are sufficiently complex that it is unlikely that the examinees could find a strategy to take advantage of the system. But the possibilities for legal difficulties are considerable under circumstances in which the test and scoring system must be disclosed. On the other hand, the quality of measurement for research purposes and in non-disclosed tests should be improved by these "bizarre" features; that is, after all, where the multiple category model obtains its additional information.

Conclusion

Item response models for multiple choice items have come of age: they fit the data. Questions remain about uses for these models. They produce excellent item analysis, but it is complex and best-represented graphically -- that breaks with the tradition of item analysis with a

few numerical summaries. Multiple category scoring clearly increases the information obtained in scoring a test -- but again at the cost of complexity, and, potentially, controversy. The validity of test scored with such methods has not been examined here, nor can it be with internal consistency data in any event. Consideration of all of these matters is deferred to future work.

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Footnotes

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Table 1.

Cross classification of responses to two items (22 and 23) of the ASVAB Form 8A General Science subtest for 1048 examinees of the NORC national probability sample (Bock and Mislevy, 1981).

		Item 2				
		A	B	C*	D	
	A	20(22)	107(23)	79(22)	44(31)	250
Item	B*	32(36)	157(34)	143(40)	35(25)	367
1	C	7(8)	36(8)	24(7)	19(14)	86
	D	30(33)	161(35)	112(31)	42(30)	345
		89	461	358	140	1048

*=Correct;

Column percentages in parentheses.

Table 2.

G^2 values for selected models for the four-item examples.

		Word Knowledge		General Science	
Model	d.f.	G^2	p	G^2	p
Bock (1972)	231	307	<.01	277	<.02
Samejima (1979)	223	271	<.02	264	<.05
"ABCD"	220	258	<.04	262	<.02
"ABCD(C),ABCD(D)"	217	248	>.05	---	--
"11 per item"	211	245	<.05	---	--
"Correct vs. incorrect"	222	---	--	253	=.05
"Correct vs., each item"	3	---	--	253	=.05

Table 3.

Estimated parameters for the last four Word Knowledge items.

Estimates for four items above, whole test below.

Parameters for correct response underscored.

		Response				
		DK	A	B	C	D
Item						
32	a_k	-1.1	-2.9	1.5	0.2	2.2
		-1.7	-1.0	1.1	0.3	1.9
	c_k	0.1	-3.8	2.9	-1.9	2.7
		0.3	-2.3	2.4	-2.5	2.1
	d_k		0.1	0.2	0.3	0.4
			0.25	0.25	0.25	0.25
33	a_k	-3.0	-0.3	1.2	2.8	-0.7
		-2.1	-0.6	1.2	2.3	-0.8
	c_k	0.1	0.6	-1.6	0.1	0.8
		2.1	0.5	-3.0	-0.6	1.0
	d_k		0.2	0.2	0.5	0.1
			0.2	0.2	0.4	0.2
34	a_k	-0.8	-1.8	-0.1	1.9	0.7
		-1.3	-0.9	-0.2	1.9	0.5
	c_k	-1.6	-3.2	0.2	2.6	2.0
		-0.9	-2.5	-0.1	1.8	1.6
	d_k		0.2	0.2	0.5	0.1
			0.2	0.2	0.4	0.2
35	a_k	-1.6	-0.4	-0.2	-0.8	3.0
		-1.9	0.5	0.0	-0.6	1.9
	c_k	0.0	-1.3	0.4	0.4	0.5
		-0.1	-2.0	0.5	0.8	0.8
	d_k		0.1	0.2	0.3	0.4
			0.25	0.25	0.25	0.25

Table 4.

Estimated parameters for the last four General Science items.

Estimates for four items above, "physical science" subset below.

Parameters for correct response underscored.

		Response				
		DK	A	B	C	D
Item 22	a_k	-2.3	-0.2	2.0	0.9	-0.3
	c_k	0.5	0.7	-0.5	-1.9	1.1
	d_k		0.2	0.4	0.2	0.2
23	a_k	-0.8	0.6	-0.5	1.1	-0.4
		-4.0	1.2	1.1	2.5	-0.9
	c_k	1.6	-2.8	1.5	0.0	-0.3
		-0.8	-0.7	1.4	0.2	-0.1
	d_k		0.2	0.2	0.4	0.2
			0.2	0.3	0.3	0.2
24	a_k	-0.5	-0.2	2.0	-1.2	0.0
		-3.3	0.2	2.6	-0.1	0.5
	c_k	-0.3	0.7	-1.0	0.7	0.0
		-1.8	1.1	-1.0	1.4	0.2
	d_k		0.2	0.4	0.2	0.2
			0.2	0.4	0.2	0.2
25	a_k	-1.5	-0.7	-0.2	0.1	2.3
	c_k	0.4	0.4	-0.5	0.5	-0.8
	d_k		0.2	0.2	0.2	0.4

Figure Captions

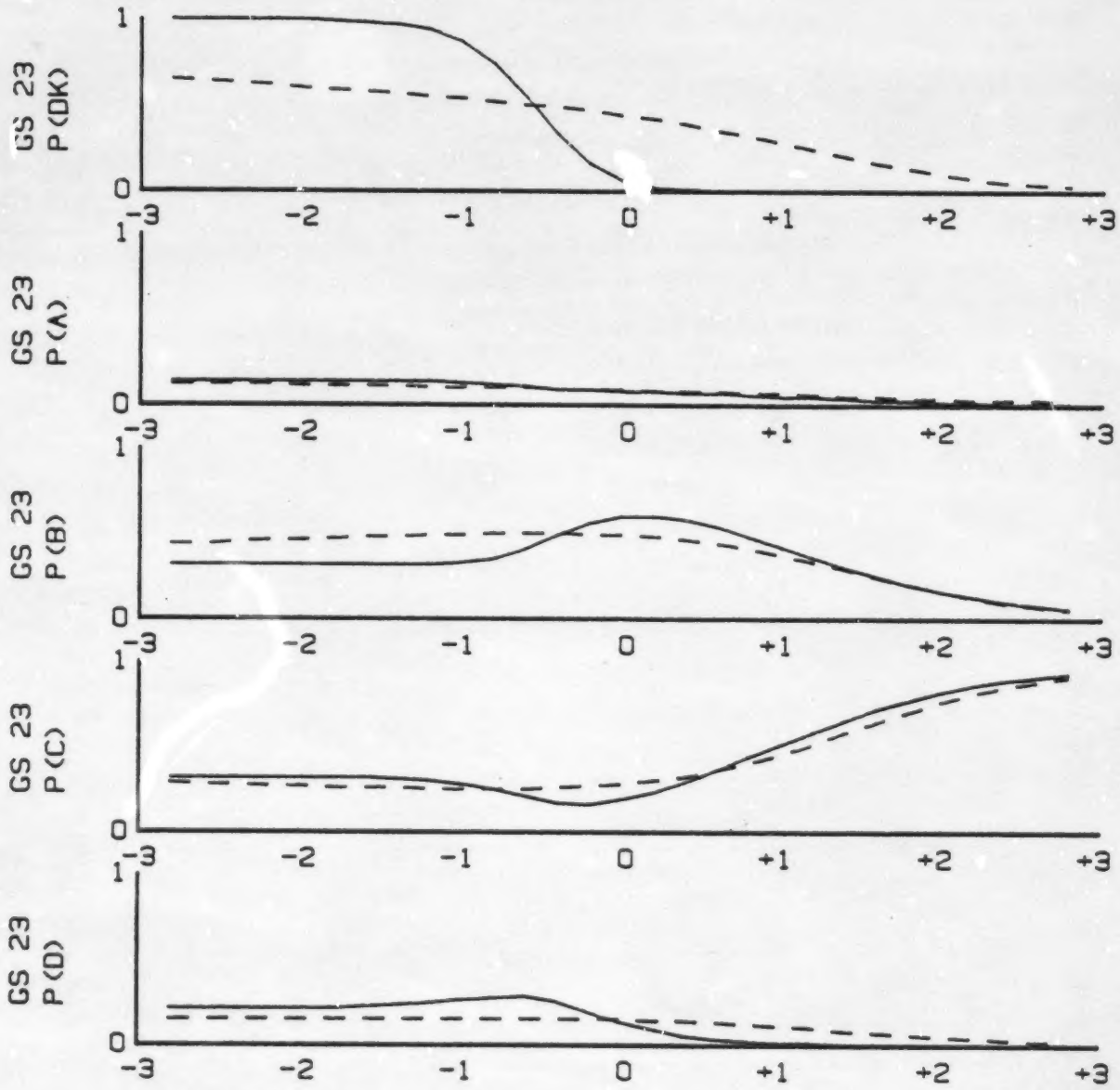
Figure 1. Five panels showing P(k), k=0,1,...,m_j as a function of θ; the solid lines correspond to the probabilities for DK, and responses A,B,C, and D for GS item 23, estimated as part of "physical science." The dashed curves use the estimates obtained with the last four items.

Figure 2. Five panels showing P(k), k=0,1,...,m_j as a function of θ; the solid lines correspond to the probabilities for DK, and responses A,B,C, and D for WK item 32 estimated in the entire test. The dashed curves use the estimates obtained with the last four items.

Figure 3. Four panels show three alternative fits of the trace line for the correct responses for the last four WK items: dots are on the standard 3-PL curve, the solid line is the model of the present paper fitted with the entire test, and the dashed line is the present model fitted with only those items.

Figure 4. Test information curves for binary (dotted line) and multiple category (solid line) scoring for WK, as functions of θ.

Figure 5. The solid lines give the trace lines from the present model and posterior density (labelled "Total") for response pattern [BBBB] (all incorrect) for the last four items of WK. Dashed lines give the corresponding curves for the same response pattern using 3-PL estimates.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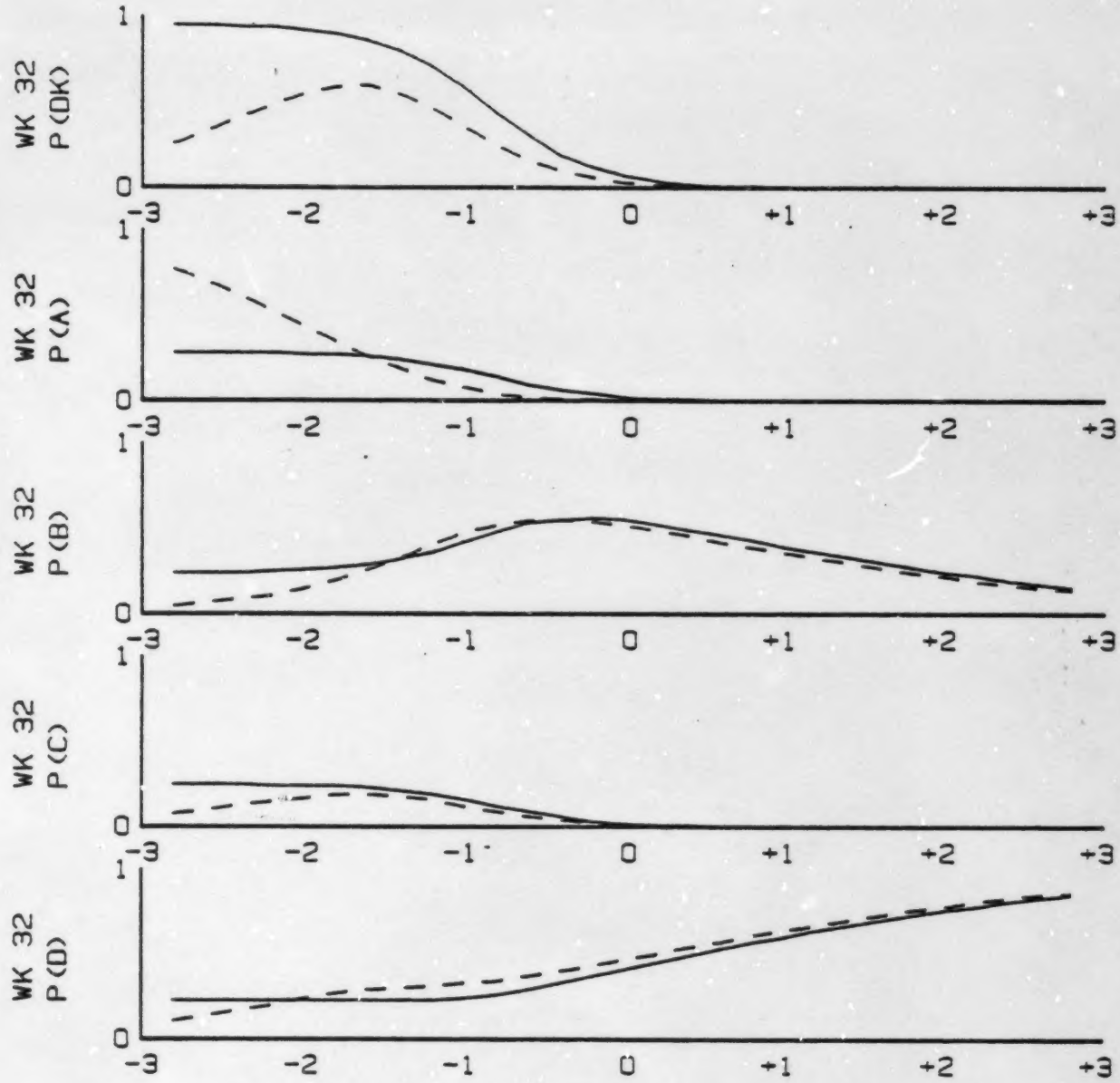
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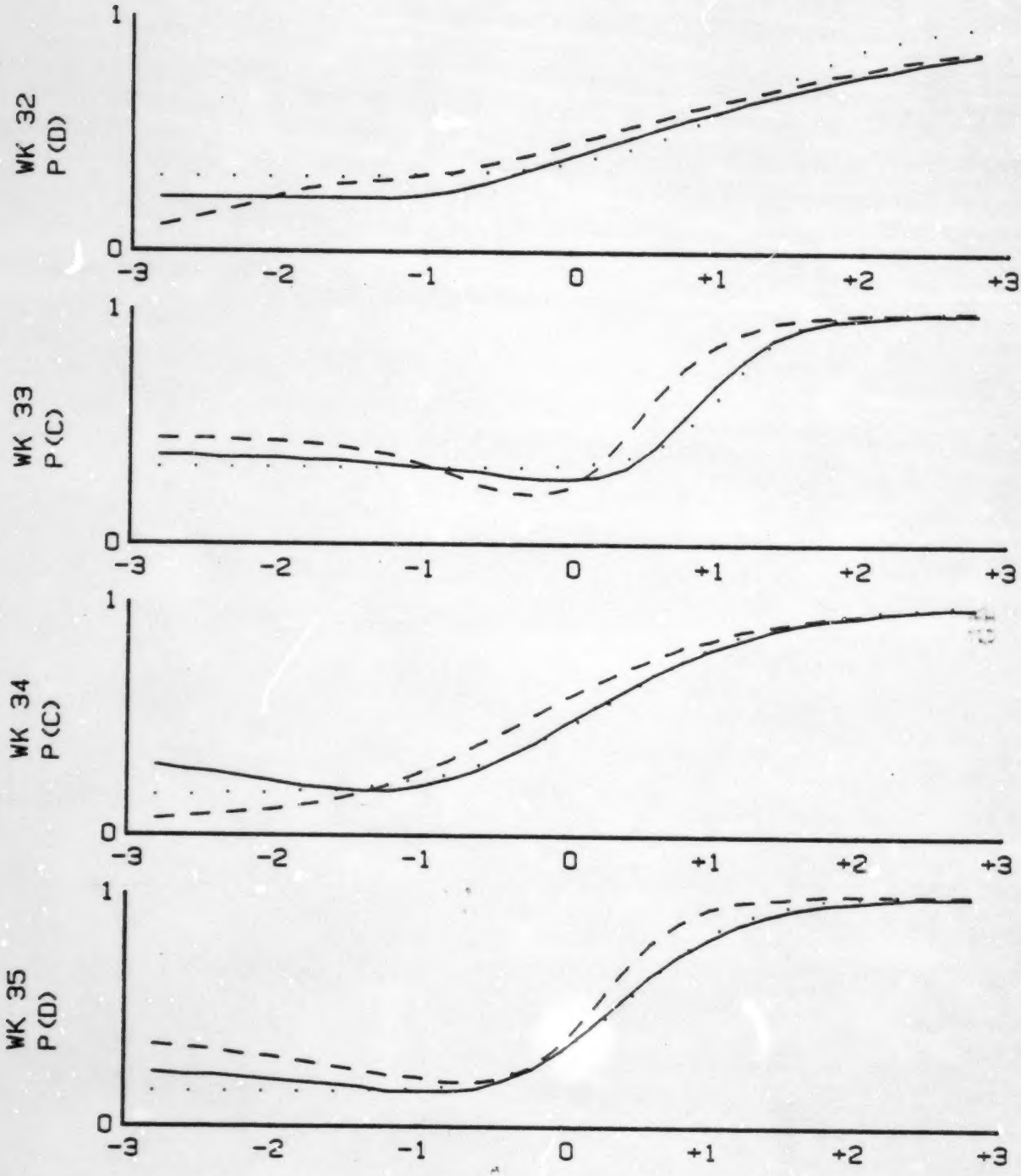
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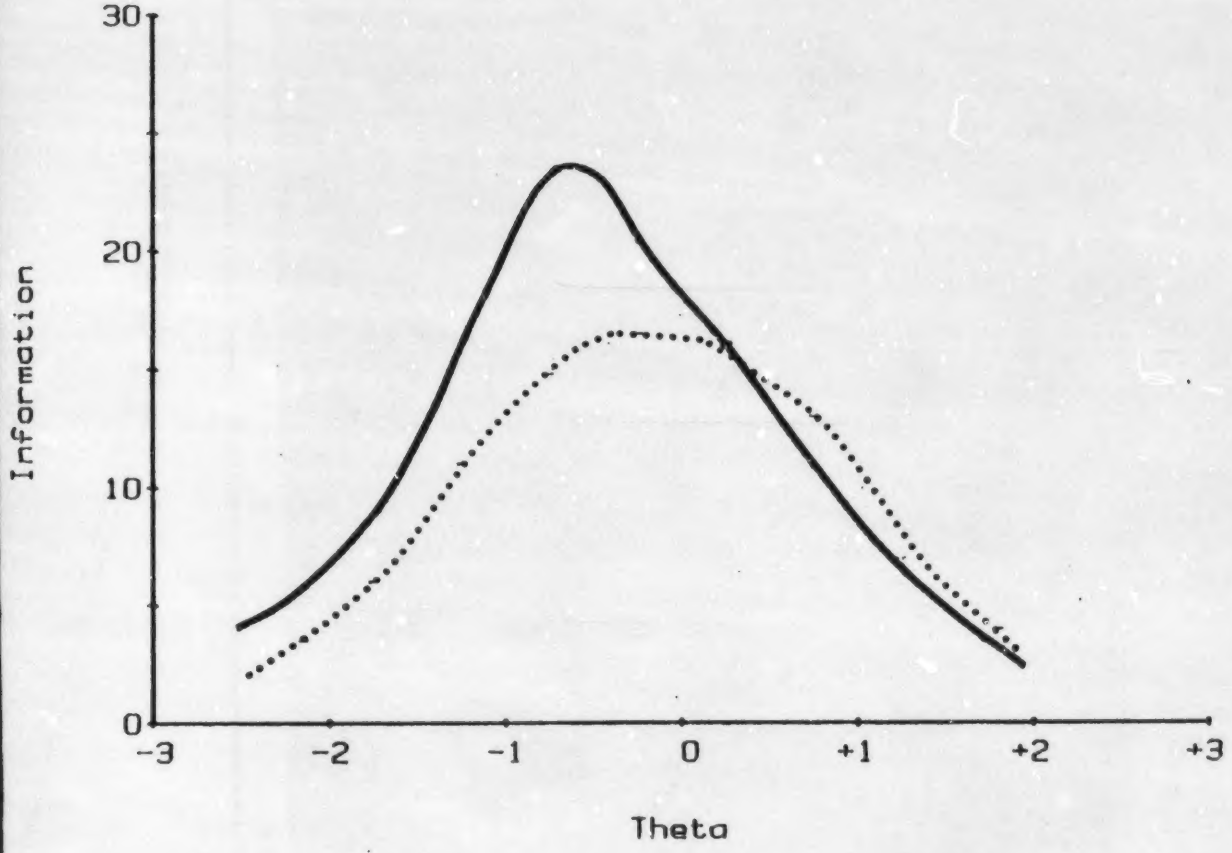
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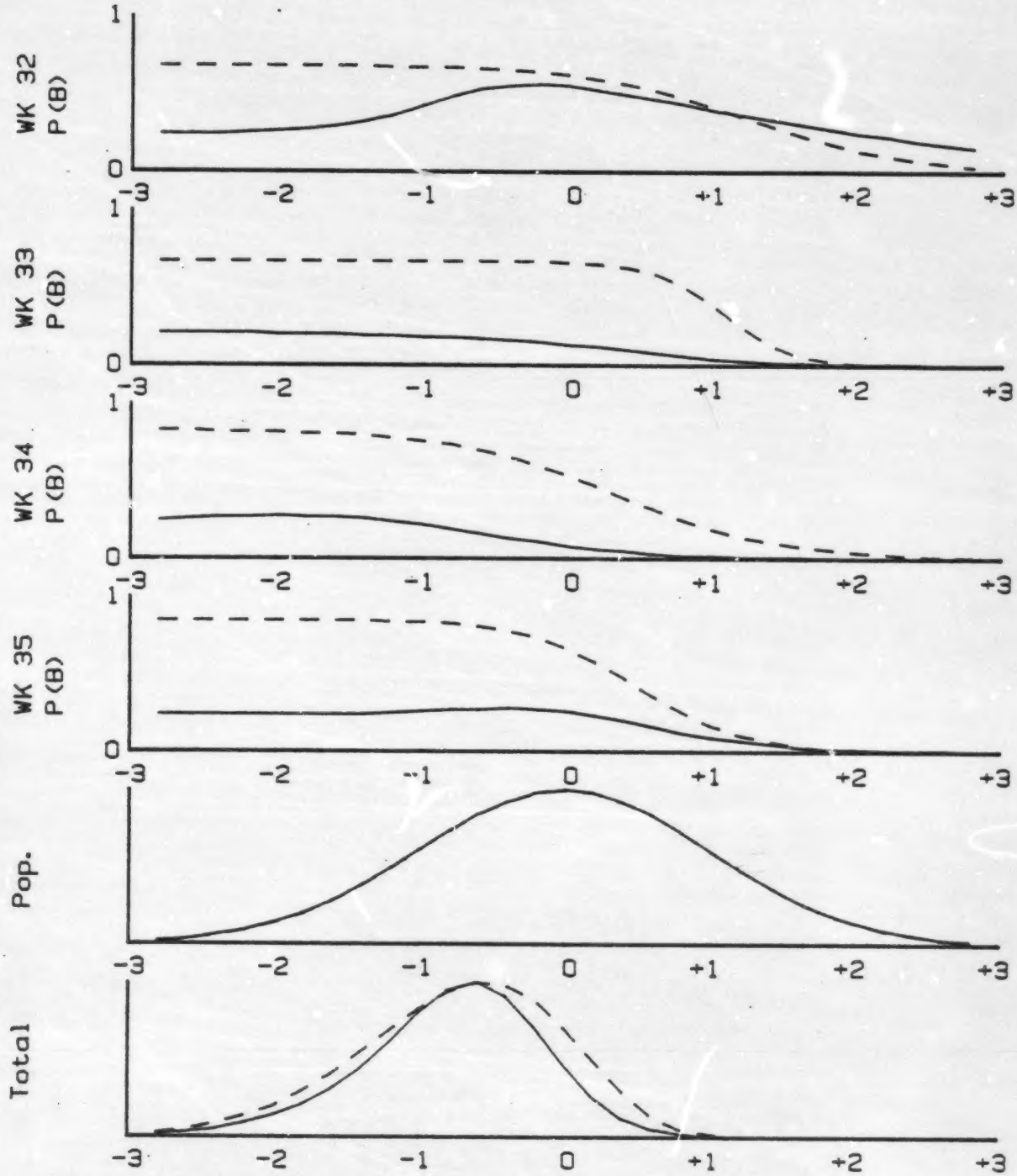
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WK 35
P(B)
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Total

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